UTEC P425/2 - MATHS 2 MARKING GUIDE

QN	SOLUTIONS	Comments
1	$P(A' \cup B) = P(A \cap B')'$	
	i.e, $\frac{2}{5} = 1 - P(A \cap B')$ (M1) $\therefore P(A \cap B') = \frac{3}{5}$ (B1)	
	$P(A) = P(A \cap B) + P(A \cap B')$	Explore other approaches.
	$= \frac{3}{10} + \frac{3}{5}$ (M1)	
	$= \frac{9}{10}$	
	$P(B/A) = \frac{P(A \cap B)}{P(A)}$ (M1)	
	$= \frac{3/10}{9/10} = \frac{1}{3}$ (A1)	
2.	Average vel. $8 = \frac{u+12}{2} \Rightarrow u = 4ms^{-1}$ (B1)	
	(i) distance, s = Average speed x time	
	$= 8 \times 4$ (M1)	
	$= 32\,m$ (A1)	Allow other methods.
	(ii) Acceleration $= \frac{v-u}{t}$	
	$= \frac{12-4}{4}$ (M1)	
	$= 2ms^{-2}$. (A1)	

QN	SOLUTIONS	Comments
3.	$x_{max} = 4.85$; $y_{max} = 3.255$	
	$x_{min} = 4.75$; $y_{min} = 3.245$ B1	
	$Min(x-y) = x_{min} - y_{max}$; $Max(x-y) = x_{max} - y_{min}$	
	$= 4.75 - 3.255$ M1 ; $= 4.85 - 3.245$	
	$= 1.495$ B1 ; $= 1.605$ B1	
	The required interval is $[1.495, 1.605]$ A1	
	Alternatively: Approx. Value $= 4.8 - 3.25$	
	$= 1.55$ B1	
	Max. error in $x-y = 0.05 + 0.005$ M1	
	$= 0.055$ B1	
	Interval $= 1.55 \pm 0.055$ M1	
	$= [1.495, 1.605]$ A1	

4 Let X = the number of men picked.

$\sim B(5, 0.7)$

$$P(X \geq 2) = 1 - P(X \leq 1) \quad M1 \quad B1$$
$$= 1 - \{P(X=0) + P(X=1)\}$$
$$= 1 - \{{}^5C_0 (0.7)^0 (0.3)^5 + {}^5C_1 (0.7)(0.3)^4\} \quad B1$$
$$= 1 - (0.00243 + 0.02835) \quad B1$$
$$= 0.96922 \ (CAL) \quad A1$$

Alternatively:

$$P(X \geq 2) \text{ at } p = 0.7 \Leftrightarrow P(X \leq 3) \text{ at } p = 0.3 \quad M1$$ Symmetry Property

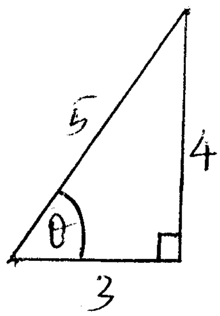
$$\Leftrightarrow 1 - P(X \geq 4) \text{ at } p = 0.3 \quad M1 \quad B1$$
$$= 1 - 0.0308 \quad B1$$
$$= 0.9692 \ (TAB) \quad A1$$

5. (i) $T = \dfrac{2u\sin\theta}{g}$

$$= \frac{2 \times 20 \times 4/5}{9.8} \quad M1$$
$$= \frac{160}{49} \text{ seconds}$$
$$\simeq 3.265306 s. \quad A1$$

(ii) $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$$t = \frac{2u\sin\theta}{3g} \Rightarrow y = \frac{2u^2\sin^2\theta}{3g} - \frac{2u^2\sin^2\theta}{9g} \quad M1$$
$$= \frac{4u^2\sin^2\theta}{9g} \quad B1$$
$$= \frac{4 \times 400 \times 16}{45 \times 49}$$
$$\simeq \underline{11.6100 \text{ m}} \quad A1$$



$\sin\theta = 4/5$

Accept other methods

QN | SOLUTIONS | Comments

6

R_{ECON}	R_{MATHS}	d	d^2
1	2	−1	1
2	4	−2	4
3	3	0	0
4	5.5	−1.5	2.25
5	1	4	16
6	5.5	−0.5	0.25
7	7	0 B1	0
			$\Sigma d^2 = 23.5$ B1

$$\rho = 1 - \frac{6 \times 23.5}{7 \times 48} \quad B_1$$

$= 0.58$; A1 ; correlation is moderate and positive.

7. Let $x = \sqrt{3} \Rightarrow x^2 - 3 = 0$ M1

$\Rightarrow f(x) = x^2 - 3 \Rightarrow f'(x) = 2x$

$$x_{n+1} = x_n - \frac{(x_n^2 - 3)}{2x_n} \quad B_1$$

$$= \frac{1}{2}\left(x_n + \frac{3}{x_n}\right);\ n = 0, 1, 2, \ldots$$

Use $x_0 = 1.5$ B1 since $1 < \sqrt{3} < 2$

$$\Rightarrow x_1 = \frac{1}{2}\left(1.5 + \frac{3}{1.5}\right)$$

$= 1.75$; $|x_1 - x_0| = 0.25$

$$x_2 = \frac{1}{2}\left(1.75 + \frac{3}{1.75}\right) \quad M_1$$

$\simeq 1.732143$; $|x_2 - x_1| = 0.017857$

$$x_3 = \frac{1}{2}\left(1.732143 + \frac{3}{1.732143}\right)$$

$\simeq 1.732051$

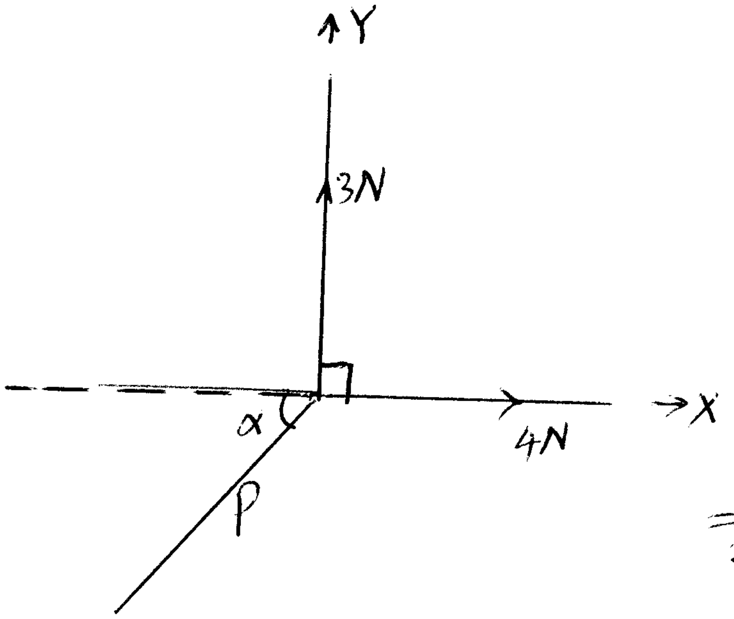
$|x_3 - x_2| = 0.000092$

$x_4 \simeq 1.732051$ A1

Thus $\sqrt{3} \simeq 1.7321$ (4dps)

4N | SOLUTIONS | Comments

8



$(\rightarrow)\ P\cos\alpha = 4$

$(\uparrow)\ P\sin\alpha = 3$ (M1)

Squaring: $P^2(\cos^2\alpha + \sin^2\alpha) = 4^2 + 3^2$ (M1)

$\Rightarrow P^2 = 25 \therefore P = 5N$ (A1)

$\frac{P\sin\alpha}{P\cos\alpha} = 3/4 \Rightarrow \tan\alpha = 3/4$ (M1)

$\alpha \simeq 36.87°$

Thus, $\theta = 90° + \alpha$
$= 126.27°$ (A1)

Lami's Theorem can be used.

SECTION B (60 Marks)

9 (a) Let x, y be the exact values $\Rightarrow \Delta X = x - X \Rightarrow x = X + \Delta X$

$\Delta Y = y - Y \Rightarrow y = Y + \Delta Y$

Error in $\frac{X}{Y} = \frac{X+\Delta X}{Y+\Delta Y} - \frac{X}{Y}$ (M1)

$= \frac{XY + Y\Delta X - XY - X\Delta Y}{Y^2(1+\frac{\Delta Y}{Y})}$

$= \frac{Y\Delta X - X\Delta Y}{Y^2(1+\frac{\Delta Y}{Y})}$ (M1)

$= \frac{Y\Delta X - X\Delta Y}{Y^2}$ (B1)

$= \frac{\Delta X}{Y} - \frac{X\Delta Y}{Y^2}$

$= \frac{X}{Y}\left[\frac{\Delta X}{X} - \frac{\Delta Y}{Y}\right]$ (M1)

$\leq \left|\frac{X}{Y}\right|\left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{-\Delta Y}{Y}\right|\right\}$ (M1)

Assumption

$\Delta Y << Y$ (B1)

$\Rightarrow \frac{\Delta Y}{Y} \simeq 0$

Hence maximum error is

$\left|\frac{X}{Y}\right|\left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta Y}{Y}\right|\right\}$ (B1)

QN	SOLUTIONS	Comments

9 (b) Max error in $(X-Y) = |\Delta x| + |\Delta y|$; $X-Y = 1.24$

$= 0.55$ B1

Max. error in $(X+Y) = 0.55$ B1 ; $X+Y = 8.36$

Hence maximum error $= \frac{1.24}{8.36}\left\{\frac{0.55}{1.24} + \frac{0.55}{8.36}\right\}$ M1B1

$\simeq 0.0755$ (4 dps) A1

Accept the Simple Interval Arithmetic Method

10

Height	0−50	50−90	90−100	100−120	120−160
freq.	8	16	20	32	4
f. density	0.16	0.4	2	1.6	0.1
c. frequency	8	24	44	76	80

B1 B1

(a) From the Histogram, mode $\simeq 98$ (see graph)

(b)

Height	50	80	90
c.f.	8	n_1	24

$\Rightarrow \frac{n_1 - 8}{24 - 8} = \frac{80 - 50}{90 - 50}$ M1

$n_1 = 8 + \frac{16 \times 30}{40}$

$= 20$ B1

Height	100	116	120
c.f.	20	n_2	32

$\Rightarrow \frac{n_2 - 20}{12} = \frac{116 - 100}{12}$

$n_2 = 20 + 16$

$= 36$ B1

The required no of pupils $= n_2 - n_1$ M1

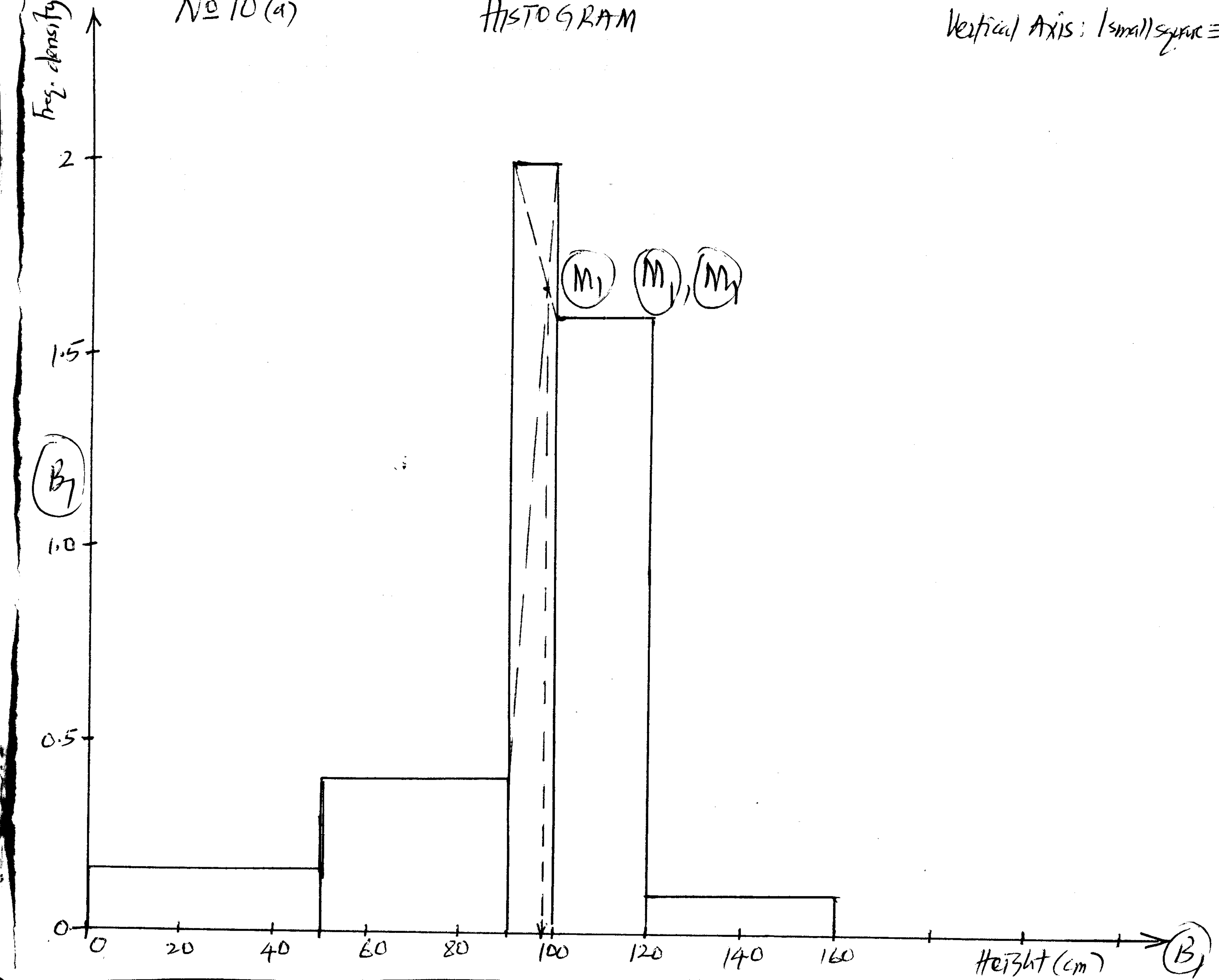
$= 36 - 20$

$= 16.$ A1

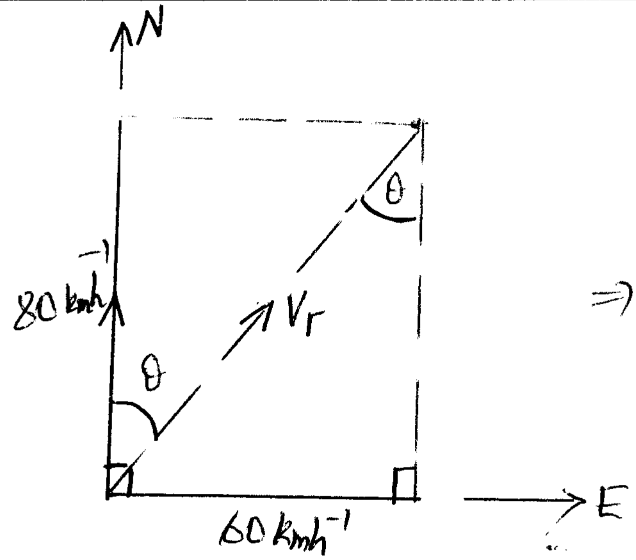
No 10 (a)

HISTOGRAM

Vertical Axis: 1 small square ≡ 0.025



QN	SOLUTIONS	Comments.
11	(a)	



$V_r^2 = 60^2 + 80^2$

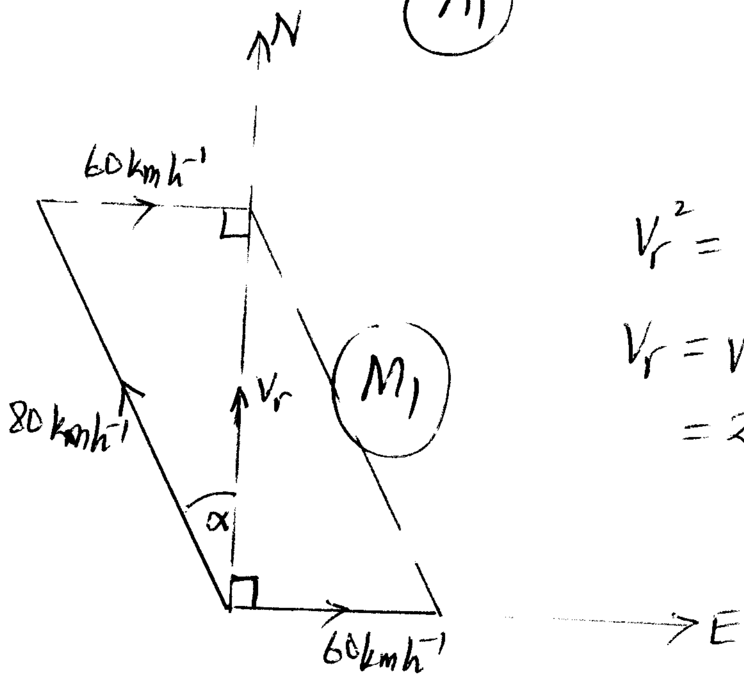
$\Rightarrow V_r = 100 \text{ kmh}^{-1}$ (M1)

$\tan\theta = \frac{60}{80}$ (M1)

$\theta \simeq 36.87°$ (B1)

The resultant vel. is 100 kmh^{-1} (A1) due N$36.87°$E. (A1)

(b)



(M1)

$V_r^2 = 80^2 - 60^2$

$V_r = \sqrt{80^2 - 60^2}$ (M1)

$= 20\sqrt{7} \text{ kmh}^{-1}$ (B1)

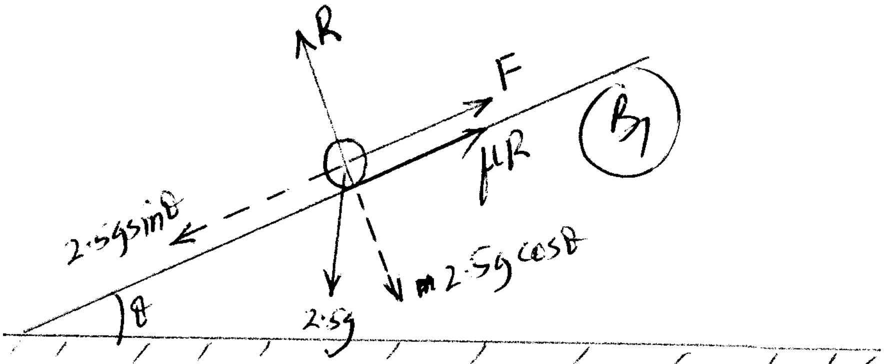
(M1) $\sin\alpha = \frac{60}{80} \Rightarrow \alpha = 48.59°$ (B1)

The required direction is N$48.59°$W (A1), with a resultant speed of $20\sqrt{7} \text{ Km h}^{-1}$. (A1)

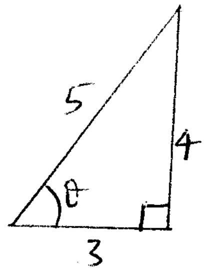
QN	SOLUTIONS	Comments
12	(a) $X \sim$ no of malaria patients.	
	$\sim B(10, 0.75)$ (B1)	
	$P(4<X<9) = P(X \leq 8) - P(X \leq 4)$; $p = 0.75$ (M1)	
	$= P(X \geq 2) - P(X \geq 6)$ (M1) $p = 0.25$	Symmetry property.
	$= 0.7560 - 0.0197$ (B1)	
	$= 0.7363$ (TAB) (A1)	
	(b) $X \sim B(48, 0.75)$; n is large (B1)	
	$X \sim N(\mu, \sigma^2)$; $\mu = 48 \times 0.75 = 36$ (B1) ; $\sigma = \sqrt{36 \times 0.25} = 3$ (B1)	
	(i) $P(X=4) \Rightarrow P(3.5 < X < 4.5)$	
	$= P\left(\frac{3.5-36}{3} < Z < \frac{4.5-36}{3}\right)$ (M1)	
	$= 0.0000$ (4 dps)	
	(ii) $P(X \leq 26) = P(X \leq 26.5)$	
	$= P\left(Z \leq \frac{26.5-36}{3}\right)$ (M1)	
	$= P(Z \leq -3.167)$ (B1)	
	$= \phi(3.167)$ (B1)	
	$= 0.0000$ (4 dps)	

QN	SOLUTIONS	COMMENTS

13 (a)



(B1)



$\tan\theta = 4/3$

$\Rightarrow \sin\theta = 4/5$ (B1)

$\cos\theta = 3/5$

Let F be the minimum force:

$R = 2.5g\cos\theta$ (B1) and $F = 2.5g\sin\theta - \mu R$ (M1)

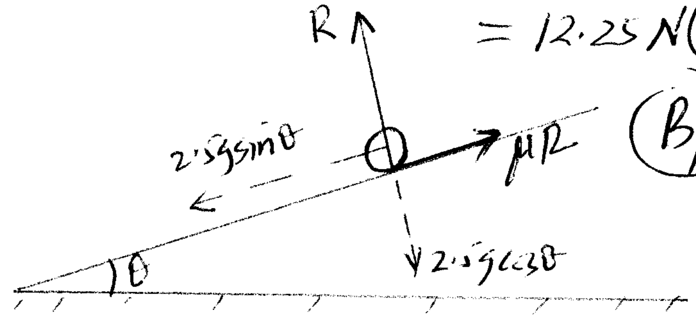
M1

$= 2.5g\sin\theta - \frac{1}{2} \times 2.5g\cos\theta$

$= 2.5 \times 9.2\left(\frac{4}{5} - \frac{1}{2} \times \frac{3}{5}\right)$ (B1)

$= 12.25\,N$ (A1)

(b)



(B1)

$\Rightarrow R = 2.5g\cos\theta$ (B1); resultant force $= 2.5g\sin\theta - 2.5\mu g\cos\theta$ (B1)

$$\text{Acceleration} = \frac{2.5g(\sin\theta - \mu\cos\theta)}{2.5} \quad \text{(M1)}$$

$$= \frac{2.5 \times 9.2(0.2 - 0.3)}{2.5}$$

$= 4.9\,ms^{-2}$ (A1)

QN	Solutions	Comments.

14 (a) $f(x) = x^3 - 2x - 1$

$f(1) = 1 - 2 - 1$
$= -2$ B1

$f(2) = 8 - 4 - 1$
$= 3$

Since $f(1) < 0$ and $f(2) > 0$

$\Rightarrow 0 < x_r < 2$ B1

x	1	x_0	2
$f(x)$	-2	0	3

By linear interpolation: $\dfrac{x_0 - 1}{2-1} = \dfrac{0-(-2)}{3-(-2)}$ M1

$x_0 = 1 + \frac{2}{5}$
$= 1.4$ A1

(b) $x_{n+1} = \dfrac{x_n}{1} - \dfrac{(x_n^3 - 2x_n - 1)}{3x_n^2 - 2}$

Comments: $f(x) = x^3 - 2x - 1$

$f'(x) = 3x^2 - 2$

$\therefore x_{n+1} = \dfrac{2x_n^3 + 1}{3x_n^2 - 2}$, $n = 0, 1, 2, \ldots$

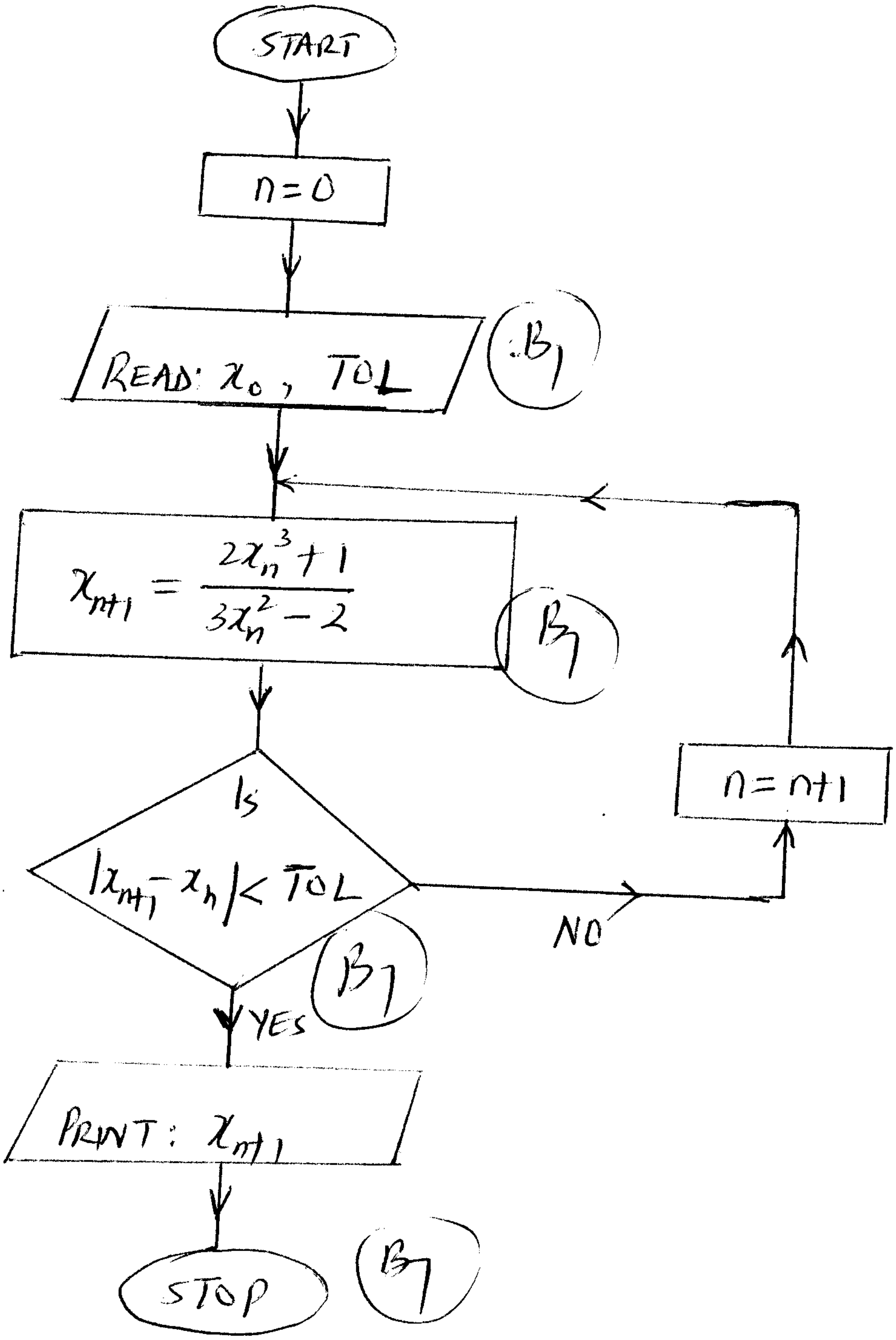
<u>Dry-Run</u>

$x_0 = 1.4$

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.4	1.6722	0.2722
1	1.6722	1.6203	0.0519
2	1.6203 B1	1.6180 B1	0.0023 B1
3	1.6180	1.6180	0.0000

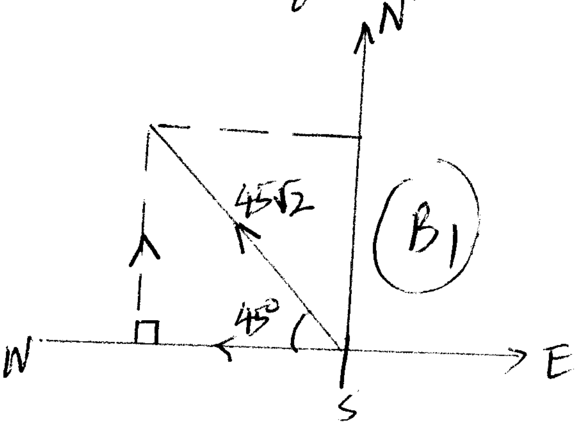
The root is 1.6180
$\simeq 1.618$ (3 dps) A1

QN	SOLNS	Comments
14(b) cont'd.	**Flow chart**	



QN | Solutions | Comments

15 (a) Velocity of Ferry



$$\underset{\sim}{V}_F = \begin{pmatrix} -45\sqrt{2}\cos 45^\circ \\ 45\sqrt{2}\sin 45^\circ \end{pmatrix} \quad (M_1)$$

$$= \begin{pmatrix} -45 \\ 45 \end{pmatrix} \text{km h}^{-1} \quad (A_1)$$

Velocity of Boat

$$\underset{\sim}{V}_B = \begin{pmatrix} 0 \\ 60 \end{pmatrix} \text{km h}^{-1} \quad (B_1)$$

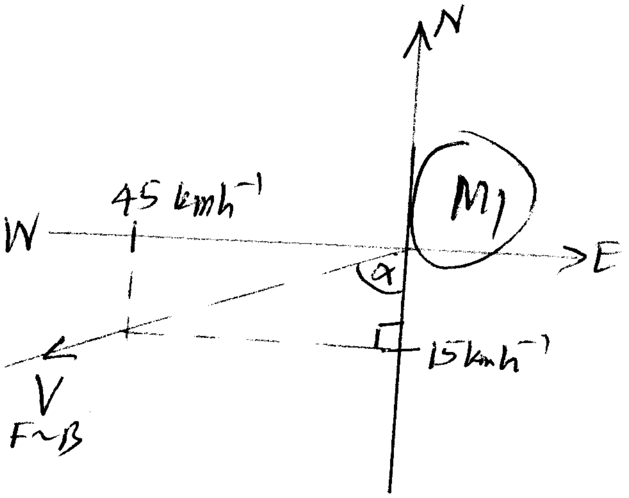
$$\Rightarrow {}_F\underset{\sim}{V}_B = \begin{pmatrix} -45 \\ 45 \end{pmatrix} - \begin{pmatrix} 0 \\ 60 \end{pmatrix} \quad (M_1)$$

$$= \begin{pmatrix} -45 \\ -15 \end{pmatrix} \text{km h}^{-1} \quad (B_1)$$

$$|{}_W V_B| = \sqrt{(-45)^2 + (-15)^2} \quad (M_1)$$

$$= 15\sqrt{10} \text{ km h}^{-1} \quad (A_1)$$

Direction of ${}_F\underset{\sim}{V}_B$



$$\tan\alpha = \frac{45}{15} = 3$$

$$\alpha \simeq 71.57^\circ \quad (B_1)$$

Hence velocity of the Ferry is $15\sqrt{10}$ kmh^{-1} due S 71.57° W (A1) (A1)

Comments:

*** Typing error

** We did not see!!

The Ferry is travelling westwards so Part (b) is unworkable.

16 Let X be the marks obtained by a candidate

$\Rightarrow X \sim N(64, \sigma^2)$

(a) $P(X > 50) = 0.60$ (M1)

$\Rightarrow P(Z > z_0) = 0.60$; where $z_0 = \dfrac{50-\mu}{\sigma}$ (M1)

From tables: $z_0 = -0.253 \Rightarrow -0.253 = \dfrac{50-64}{\sigma}$ (B1)

$\therefore \sigma = \dfrac{14}{0.253}$

$\simeq 55$ (A1)

(b) Let x_0 be the pass mark

$\Rightarrow P(X \geq x_0) = 0.75$ (M1)

$\Rightarrow P(Z \geq z_0) = 0.75$; $z_0 = \dfrac{x_0 - 64}{55}$ (B1)

$\Rightarrow -0.674 = \dfrac{x_0 - 64}{55}$

$\Rightarrow x_0 = 64 - 0.674 \times 55$ (M1)

$\simeq 27$ (A1)

(c) $P(45 < X < 55) = P\left[\dfrac{45-64}{55} < Z < \dfrac{55-64}{55}\right]$ (M1)

$= P(-0.3455 < Z < -0.1636)$

$\Leftrightarrow P(0.1636 < Z < 0.3455)$

$= 0.1353 - 0.0652$

$= 0.0701$ (A1)

No. required $= 2000 \times 0.070$ (M1)

$= 140$ (A1)